In this case, $J < 0$, i.e., the mass of the disperse phase increases. The rate of interphase mass transfer is determined in the form

$$J = -6\rho_p/(\delta\rho_p^0)a, \quad a = K_0 \exp(-E/RT_p), \tag{2.7}$$

where a is the change in mass due to oxidation of a particle per unit time per unit of its area, K_0 is a pre-exponential multiplier, E is the activation energy, and R is the universal gas constant.

From (2.7), $\gamma = a\delta/(3\mu)$. Let us evaluate the upper boundary of this ratio. At 600°C in dry air, a $0.21\cdot10^{-4}$ kg/(m$^2\cdot$sec) [7]; for the particle sizes $\delta = 10^{-5}$-10^{-4} m characteristic of power plants, the ratio $\gamma \approx 10^{-6}$-10^{-5} is so low that ε_J can be ignored in the equation for turbulence energy (1.3).

Thus, the direct effect of interphase mass transfer on turbulence energy must be considered in the case of intensive phase transformations.

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ONSET OF THERMOCAPILLARY CONVECTION IN A TWO-LAYER SYSTEM WITH THE RELEASE OF HEAT AT THE INTERFACE

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The onset of thermocapillary convection in a two-layer system with heating from below or above was studied in [1-4]. It was established that instability of the equilibrium state can result in both monotonic and oscillatory disturbances. Under certain conditions, only oscillatory instability is possible [3]. The presence of heat sources or sinks at the interface between the media – which may be due to a chemical reaction, absorption of radiation, etc. – has a significant effect on the stability of the system. The problem of the stability of the equilibrium state with surface heat release was solved in [5] in regard to monotonic disturbances.

Here, we study the effect of surface heat release and heat absorption on the stability of the equilibrium of a two-layer system in the presence of both monotonic and oscillatory instability. We will examine the evolution of oscillatory neutral curves for several characteristic cases. It is established that the heat release has a stabilizing effect on both monotonic and oscillatory disturbances.

1. Let the space between two horizontal solid plates – on which constant and different temperatures are maintained (temperature difference equal to θ) – be filled by two layers of viscous immicible fluids. The x axis is directed horizontally, while the y axis is directed

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vertically upward. The equations of the solid boundaries are $y = a_1$ and $y = -a_2$. Thermocapillary convection occurs in the presence of gravitational force, which ensures the existence of a plane interface. The effect of buoyancy on convection in this case is negligible compared to the thermocapillary effect which occurs for thin liquid films. At the interface of the media $y = 0$, assumed to be nondeformable, we assign a constant level of heat release Q_0 ($Q_0 < 0$ corresponds to heat absorption). The dynamic and kinematic viscosities, thermal conductivity, and diffusivity: $\eta_m, \nu_m, \varkappa_m, \chi_m$ (m = 1 for the upper liquid and m = 2 for the lower liquid). The surface tension is linearly dependent on temperature: $\sigma = \sigma_0 - \alpha T$.

Mechanical equilibrium is characterized by a constant value for the vertical temperature gradients A_m (m = 1, 2), which are determined from the heat-balance equation at the interface $-\varkappa_1 A_1 + \varkappa_2 A_2 = Q_0$ and the relation $A_1 a_1 + A_2 a_2 = -s\theta$ ($s = 1$ with heating from below, $s = -1$ with heating from above): $A_1 = -(s\theta\varkappa_2 + Q_0 a_2)/(a_1\varkappa_2 + a_2\varkappa_1)$, $A_2 = -(s\theta\varkappa_1 - Q_0 a_1)/(a_1\varkappa_2 + a_2\varkappa_1)$.

We introduce the notation: $\eta = \eta_1/\eta_2$, $\nu = \nu_1/\nu_2$, $\varkappa = \varkappa_1/\varkappa_2$, $\chi = \chi_1/\chi_2$, $a = a_2/a_1$. As units of length, time, the stream function, and temperature we choose a_1, a_1^2/ν_1, ν_1 and θ. The dimensionless temperature gradient in equilibrium $A_1 = -(s + Qa\varkappa)/(1 + \varkappa a)$ in the upper liquid and $A_2 = -\varkappa(s - Q)/(1 + \varkappa a)$ in the lower liquid ($Q = Q_0 a_1/\theta\varkappa_1$).

We subject the equilibrium state to perturbations of the stream function ψ_m' and temperature T_m':

$$(\psi_1', T_1', \psi_2', T_2') = (\psi_1(y), T_1(y), \psi_2(y), T_2(y)) \exp[ikx - (\lambda + i\omega)t]$$

(k is the wave number, $\lambda + i\omega$ is the complex decrement). The linearized equations for ψ_m' and T_m' have the form [4]

$$\begin{gathered}(\lambda + i\omega) D\psi_m = -d_m D^2\psi_m, \\ -(\lambda + i\omega) T_m - ik\psi_m A_m = \frac{e_m}{\Pr} DT_m \quad (m = 1, 2),\end{gathered} \tag{1.1}$$

where $D = d^2/dy^2 - k^2$, $d_1 = e_1 = 1$, $d_2 = 1/\nu$, $e_2 = 1/\chi$, $\Pr = \nu_1/\chi_1$ is the Prandtl number.

Using the prime to denote differentiation with respect to y, we write the conditions on the solid boundaries:

$$y = 1: \psi_1 = \psi_1' = T_1 = 0, \quad y = -a: \psi_2 = \psi_2' = T_2 = 0 \tag{1.2}$$

and on the interface

$$\begin{gathered}y = 0: \psi_1 = \psi_2 = 0, \ \psi_1' = \psi_2', \ T_1 = T_2, \\ \varkappa T_1' = T_2', \ \eta\psi_1'' - ik \operatorname{Mr} T_1 = \psi_2''\end{gathered} \tag{1.3}$$

($\operatorname{Mr} = \eta M/\Pr$, $M = \alpha\theta a_1/\eta_1\chi_1$ is the Marangoni number). The boundary of instability is determined by the condition $\lambda = 0$.

2. Boundary-value problem (1.1)-(1.3) has an analytical solution [5] for the case of monotonic instability ($\lambda = \omega = 0$). The expression for the critical number Mr in our notation has the form

$$\operatorname{Mr}(k) = \frac{8k^2(1 + \varkappa a)(\varkappa D_1 + D_2)(\eta B_1 + B_2)}{\varkappa \Pr[s(\chi C_2 - C_1) - Q(\chi C_2 + a\varkappa C_1)]}, \tag{2.1}$$

where

$$B_1 = \frac{s_1c_1 - k}{s_1^2 - k^2}; \ B_2 = \frac{s_2c_2 - ka}{s_2^2 - k^2a^2}; \ C_1 = \frac{s_1^3 - k^3c_1}{s_1^2 - k^2}; \ C_2 = \frac{s_2^3 - k^3a^3c_2}{s_2^2 - k^2a^2};$$

$$D_1 = c_1/s_1; \ D_2 = c_2/s_2; \ s_1 = \operatorname{sh} k; \ s_2 = \operatorname{sh} ka; \ c_1 = \operatorname{ch} k; \ c_2 = \operatorname{ch} ka.$$

In analyzing the effect of the surface heat release on monotonic stability, it is convenient to introduce the parameter

$$\operatorname{Mr}_Q = \operatorname{Mr}\, Q = \frac{\alpha Q_0 a_1^2}{\eta_2\nu_1\varkappa_1}. \tag{2.2}$$

In contrast to Q, the parameter Mr_Q is independent of θ and remains constant with a change in the temperature difference between the upper and lower boundaries of the system. Different values of Mr_Q correspond to different rates of heat release at the interface. In the new variables, Eq. (2.1) is written as

$$\mathrm{Mr}(k) = s\,\frac{8k^2(1+\varkappa a)(\mathrm{Pr}\,\varkappa)^{-1}(\varkappa D_1 + D_2)(\eta B_1 + B_2) + \mathrm{Mr}_Q(\chi C_2 + a\varkappa C_1)}{(\chi C_2 - C_1)}. \tag{2.3}$$

It is evident that heat release at the boundary ($\mathrm{Mr}_Q > 0$) always stabilizes the monotonic mode of instability, while heat absorption ($\mathrm{Mr}_Q < 0$) destabilizes it. This effect can be understood on the basis of qualitative arguments. The appearance of a hot spot at the boundary leads to inflow of liquid from the direction of the solid boundaries and its spreading over the interface. If the interface is heated relative to the solid boundaries, then the resulting inflow of colder liquid leads to decay of the temperature perturbation. If the interface is cooled, then the inflow of warmer liquid intensifies the temperature perturbation.

Let us discuss the special case $\chi = 1$, $a = 1$. It was established in [3] that there is no monotonic instability without the heat release. As can be seen from Eq. (2.3), when heat is released the boundary of monotonic instability exists:

$$\mathrm{Mr}_Q = -\frac{8k^2(1+\varkappa)(1+\eta)}{\mathrm{Pr}\,\varkappa}\,\frac{s_1c_1 - k}{s_1^2 t_1 - k^3} \quad (t_1 = s_1/c_1) \tag{2.4}$$

and is independent of Mr. The problem must be solved numerically to obtain the boundaries of oscillatory instability.

We will examine a system with the parameters $\eta = \nu = 0.5$; $\varkappa = \chi = \mathrm{Pr} = a = 1$. We will restrict ourselves to the case of heating from below. Monotonic instability occurs at $\mathrm{Mr}_Q < \mathrm{Mr}_{Q*} < 0$, where Mr_{Q*} is found from the extremum of Eq. (2.4). At $\mathrm{Mr}_Q > \mathrm{Mr}_{Q*}$ (in particular, in the absence of heat absorption), oscillatory instability is the only possible mechanism by which the equilibrium state could become unstable. To analyze the effect of heat release on convective stability, we will calculate neutral curves for fixed Q (Fig. 1). At $Q > 0$, the neutral curve stabilizes with an increase in Q; no monotonic neutral curve appears. Conversely, at $Q < 0$, with an increase in [Q] the oscillatory neutral curve shifts to the region of smaller Mr. Moreover, a monotonic neutral curve does appear at $|\mathrm{Mr}_Q| = \mathrm{Mr}|Q| > |\mathrm{Mr}_{Q*}|$. Figure 1 shows oscillatory (dashed lines) and monotonic (solid lines) neutral curves constructed for the following Q: 0) line 1; 2) 0.015; 3) 0.03; 4, 5) −0.02; 6, 7) −0.025; 8, 9) −0.03. With an increase in $|Q|$ ($Q < 0$), the monotonic mode of instability destabilizes the equilibrium state less intensively than the oscillatory mode. Figure 2 shows graphs of the dependence of the frequency of oscillation on the wave number for Q = 0; 0.015; 0.03; −0.02; −0.025; −0.03 (lines 1-6). Figure 3 shows the dependence of values of Mr_*, minimized with respect to k, on Mr_Q for the oscillatory (line 1) and monotonic (2) modes of instability. The oscillatory mode is most dangerous at $\mathrm{Mr}_Q > \mathrm{Mr}_Q^*$, while monotonic disturbances are the most dangerous in the region $\mathrm{Mr}_Q < \mathrm{Mr}_Q^*$.

Now let us examine a system of real liquids, consisting of transformer oil and formic acid. The system has the following parameters: $\eta = 11.1$, $\nu = 15.4$, $\varkappa = 0.41$, $\chi = 0.714$, $\mathrm{Pr} = 306$, $a = 1.667$. In the absence of heat release ($Q = 0$), the system is unstable against monotonic during heating on the side of both the first liquid and the second liquid (line 1 in Fig. 4). In addition, oscillatory instability may develop in the longwave region (line 2). At $Q > 0$, an increase in Q is accompanied by stabilization of all fragments of the

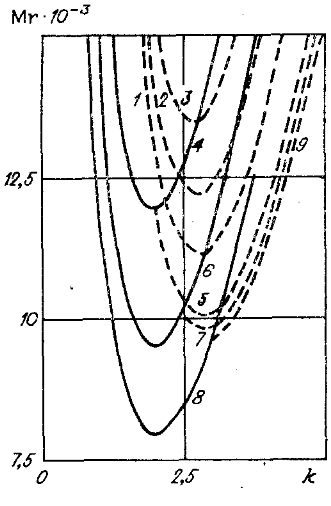

Fig. 1

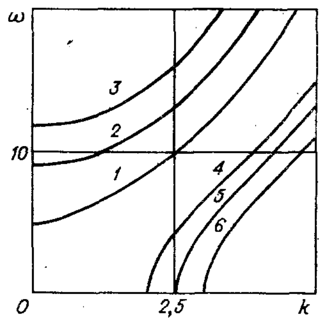

Fig. 2

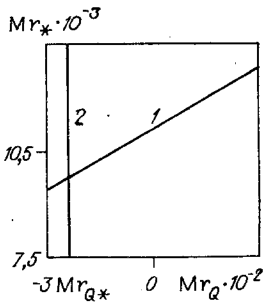

Fig. 3

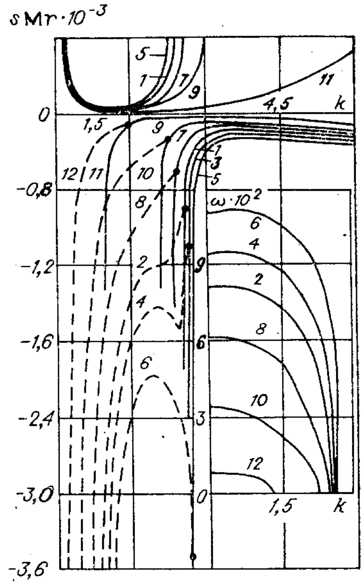

Fig. 4

neutral curve: Q = 0.015 (lines 3 and 4); 0.03 (5, 6). The fragment of the line 3 for s = 1 is not presented due to the proximity to line 1 in the scale of the graph. At Q < 0, destabilization takes place: Q = −0.03 (lines 7 and 8); −0.09 (9, 10); −0.3 (11, 12). Throughout the investigated region of Q, the minimum of the neutral curve is realized for monotonic perturbations. The insert in Fig. 4 shows the dependence of ω on k for oscillatory perturbations (the numeration of the lines for the insert corresponds to the numeration of the lines in the main part of the graph).

Thus, the conclusion reached regarding the stabilizing effect of heat release and the destabilizing effect of heat absorption is valid not only for monotonic perturbations but also oscillatory perturbations.

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